বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. Honours Examinations 2020

(Under CBCS Pattern)

Semester - V

Subject: PHYSICS

Paper: C11T & C11P

(Quantum Mechanics and Applications)

Full Marks : 60

Time : 3 Hours

Candiates are required to give their answer in their own words as far as practicable.

The figures in the margin indicate full marks.

Group - A

THEORY (Marks : 40)

Answer any ***two*** from the following questions : 2×20

1. Answer any ***five*** questions from the following : 5×4=20

(a) What is Gyromagnetic ratio for the orbital and spin motion of an electron in an atom.

(b) State Paul's exclusion principle. Give electronic configuration for the element. Ni ($Z = 28$).

(c) Find the precessional frequency of an electron orbit when placed in magnetic field of 6 Wb/m^2.

(d) Calculate the value of lowest energy of an electron in a one dimensional force free region of length 4A°.

(e) Distinguish between a classical and quantum harmonic oscillator.

(f) Calculate the normalization constant for a wave function (at $t = 0$) given by $\Psi(x) = ae^{-a^2x^2/2}e^{ikx}$ known as a Gaussian wave packet.

(g) What is Larmor's theorem ?

2. (a) (i) Using Heisenberg uncertainty principle, calculate the ground state energy and radius of hydrogen and helium atom.

(ii) Using the energy-time uncertaity principle calculate the width of the spectral line when the atom de-exites to the gound state.

(iii) Using the uncertainty principle, show that an alpha particle can exist inside a nucleus.

5+2+3

(b) (i) State and prove Ehrenfest's theorem.

(ii) What are the continuity and boundary conditions that must be satisfied for a wave function to be physically acceptable ?

(iii) Show that the uncertainty relation $\Delta x \Delta p \geq \hbar / 2$ is satisfied in the case of particle in a one-dimensional box. $\left(\hbar = h/2\pi\right)$

(iv) Calculate the probability that a particle in a one-dimensional box a length L can be found between 0.4 L to 0.6 L for the (a) ground state, (b) first excited state, (c) second excited state. 3+2+3+2

3. (a) (i) What is Anomalous Zeeman effect ? Explain with the help of diagram the transition between 3d and 2p levels in a Normal Zeeman effect. 3+2

(ii) Calculate the Lande g factor for the following states : (i) $3\,^2S_{1/2}$ (ii) $4\,^2P_{1/2}$.

2+2

(b) (i) The normalized ground state wave function of hydrogen atom is given by, $\Psi(r,\theta,\varphi)=\frac{1}{\sqrt{\pi a^{3/2}}}e^{-2r/a},\ 0<r<\infty$. Find the value of $\left\langle\frac{1}{r}\right\rangle$

(ii) Write down the Hamiltonian for the hydrogen atom.

(iii) Prove that for the hydrogen atom the wave functions Ψ_{100} and Ψ_{200} are orthogonal.

(iv) What do you mean by the degeneracy of state ? 4+2+2+2

4. (a) Define the creation (a^+) and annihilation (a) operators for a harmonic oscillator and show that $\widehat{H}=\hbar\omega\left(a^+a+\frac{1}{2}\right)$ 5

(b) A particle in the harmonic oscillator potential starts out in the state

$$\psi(x,0)=A\left[3\psi_0(x)+4\psi_1(x)\right]$$

(A) Find A (B) Construct $\psi(x,t)$ and $|\psi(x,t)|^2$ 5

(c) Suppose a particle starts out in a linear combination of just two stationary states :

$$\psi(x,0)=c1\psi_1(x)+c2\psi_2(x)$$

(i) What is the wave function $\psi(x,t)$ at subsequent times ?

(ii) Is it a stationary state ?

(iii) Discuss briefly, with theory, the Stren-Gerlach experiment. Justify the use of a bean of silver atoms in the experiment. 2+3+5

Group - B

PRACTICAL (Marks : 20)

Answer any ***one*** from the following questions : 1×20

1. Demonstrate in details the tunneling effect in tunnel diode using I-V characteristics.

2. Solve the s-wave radial Schrodinger equation for the vibrations of hydrogen molecule :

$$\frac{d^2y}{dr^2}=A(r)u(r),\ A(r)=\frac{2\mu}{h^2}\left[V(r)-E\right]$$

Where μ is the reduced mass of the two-atom system for the Morse potential

$$V(r) = D\left(e^{-2\alpha r'} - e^{-\alpha r'}\right), r' = \frac{r - r_0}{r}$$

Find the lowest vibrational energy (in MeV) of the molecule to an accuracy of three significant digits. Also plot the corresponding wave function.

Take $m = 940 \times 10^6 \text{ eV}/C^2$, $D = 0.755501 \text{ eV}$, $\alpha = 1.44$, $r_0 = 0.131349 Å$

3. Solve the s-wave radial Schrodinger equation for a particle of mass m :

$$\frac{d^2 y}{dr^2} = A(r)u(r),\ A(r) = \frac{2m}{h^2}\left[V(r) - E\right]$$

For the anharmonic oscillator potential.

$$V(r) = \frac{1}{2}kr^2 + \frac{1}{3}br^3$$

for the ground state energy (in MeV) of particle to an accuracy of three significant digits. Also, plot the corresponding wave function.

Choose $m = 940 \text{ MeV}/c^2$, $k = 100 \text{ MeV } fm^{-2}$, $b = 0, 10, 30 \text{ MeV } fm^{-3}$.

In these units, $c\hbar = 197.3$ MeV fm. The ground state energy I expected to lie between 90 and 110 MeV for all three cases.